NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BSAM	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA 2
SESSION: NOVEMBER 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

FIRST OPPORTUNITY EXAMINATION PAPER	
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INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [42]

1.1. [9]

Let V and U be vector spaces over a field $\mathbb{R}$ and let $T: V \to U$ be a mapping. Then define what does it means to say

(a) T is linear. [3]

(b) Kernel of T. [2]

(c) rank of T and nullity of T. [4]

1.2. [33]

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a mapping defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ 3x \\ y + z \end{bmatrix}$.

(a) Show that T is linear. [11]

(b) Determine Ker (T), the nullity of T and the rank of T and use the result together with the rank theorem to determine whether T is an isomorphism or not. [11]

(c) Determine the matrix representation of T with respect to the basis $\{v_1, v_2, v_3\}$ where $v_1 = (1, 0, 1), v_2 = (0, 1, 1), v_3 = (1, 1, 0)$ of $\mathbb{R}^3$. [5]

(d) Determine the determinant of T and trace of T. [6]

QUESTION 2 [10]

Let S and $\mathcal{B}$ be bases for $\mathbb{R}^2$ where $S = \left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$ and $\mathcal{B} = \left\{\begin{bmatrix} 8 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \end{bmatrix}\right\}$.

Find the change of basis matrix from $\mathcal{B}$ to S ($P_{S \leftarrow \mathcal{B}}$).

QUESTION 3 [11]

3.1. State what does it means to say two matrices are similar. [2]

3.2. Let A and B be n x n similar matrices. Then show that det A = det B. [5]

3.3. Show that $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ are not similar. [4]

QUESTION 4 [12]

Find the coordinate vector $[p(x)]_{\mathcal{B}}$ of $p(x) = 5 + 4x - 3x^2$ with respect to the basis

$\mathcal{B} = \{1 - x,\ 1 + x + x^2,\ 1 - x^2\}$ of P_2.

QUESTION 5 [15]

Let $A = \begin{bmatrix} 2 & -3 & 7 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$. Then

5.1. determine the characteristic polynomial and the eigenvalues of A. [6]

5.2. is A diagonalizable? Justify your answer. [3]

5.3. find the eigenspace corresponding to the largest eigenvalue of A. [6]

QUESTION 6 [10]

Find the quadratic form $q(x_1, x_2, x_3)$ for the symmetric matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER